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Transport driven by spatially modulated noise in a periodic tube

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Abstract
This paper investigates a three-dimensional periodic tube driven by spatially modulated Gaussian white noise. We derive an analytical expression for the net current by introducing entropic barriers. It is found that the phase shift between the entirely symmetric tube and noise modulation can break the symmetry of the generalized potential and induce directed transport. The sign of the current is determined by the phase shift. The current is a peaked function of the bottleneck radius. The interplay between the asymmetric tube and noise modulation can also induce a net current.

1. Introduction

Brownian motion in periodic structures can describe diverse process in many different branches of science. There has been an increasing interest in the transport properties of nonlinear systems which can extract usable work from unbiased nonequilibrium fluctuations [1–4]. This comes from the desire to understand molecular motors [5], nanoscale friction [6], surface smoothing [7], coupled Josephson junctions [8], optical ratchets and directed motion of laser cooled atoms [9] and mass separation and trapping schemes at the microscale [10].

The focus of research has been on noise-induced unidirectional motion. The directed Brownian motion of particles is generated by nonequilibrium noise in the absence of any net macroscopic forces and potential gradients. Diffusive motion with a state-dependent noise plays an important role in many physical systems [11]. Some examples are: nonlinear self-excited oscillators in the presence of noise, diodes, current instabilities in bulk semiconductors and in ballast resistors. State-dependent noise can break the symmetry of the generalized potential and induce a net current [12]. A bias in the generalized potential can be induced by asymmetry of the energy potential and noise modulation [13] or by a phase shift between

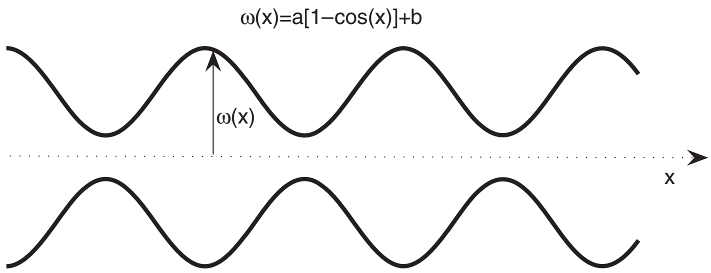

Figure 1. Schematic diagram of a tube with periodicity 2π. The shape is described by the radius of the tube $\omega(x) = a[1 - \cos(x)] + b$.

an entirely symmetric potential and noise modulation [11]. An additive temporal–spatial noise can also produce a nonzero net current even when the potential is symmetric [14].

Previous studies on state-dependent noises have considered the energy barrier. The nature of the barrier depends on which thermodynamic potential varies when passing from one well to the other, and the presence of a barrier plays an important role in the dynamics of the solid state physics system. However, in some cases, such as soft condensed matter and biological systems, entropy barriers should be considered. Entropy barriers may appear when coarsening the description of a complex system to simplify its dynamics. Reguera and co-workers [15] used the mesoscopic nonequilibrium thermodynamics theory to derive the general kinetic equation of a system and analyse in detail the case of diffusion in a domain of irregular geometry in which the presence of the boundaries induces an entropy barrier when approaching the dynamics by a coarsening of the description. In the presence of entropic barriers, the asymmetry of the tube can induce a net current in the absence of any net macroscopic forces or in the presence of unbiased forces [16].

Previous works on state-dependent noise considered the energy barriers. The present work is extended to the study of entropic barriers. Our emphasis is on finding how the phase shift between the entirely symmetric tube and noise modulation *or* the interplay between the asymmetric tube and noise modulation can induce a net current.

2. Noise-induced current in a periodic tube

Consider Brownian particles in a symmetric periodic tube (figure 1) subject to a spatially modulated Gaussian white noise. The stochastic dynamics is governed by the three-dimensional (3D) Langevin equations in a dimensionless form [15, 16]:

$$\eta \frac{\mathrm{d}x}{\mathrm{d}t} = g(x)\sqrt{\eta k_\mathrm{B} T}\xi_x(t), \tag{1}$$

$$\eta \frac{\mathrm{d}y}{\mathrm{d}t} = \sqrt{\eta k_\mathrm{B} T}\xi_y(t), \tag{2}$$

$$\eta \frac{\mathrm{d}z}{\mathrm{d}t} = \sqrt{\eta k_\mathrm{B} T}\xi_z(t), \tag{3}$$

where x, y, z are coordinates, T the temperature, k_B the Boltzmann constant and η the friction coefficient of the particle. $\xi_{x,y,z}(t)$ is Gaussian white noise with zero mean and correlation function $\langle \xi_i(t)\xi_j(t')\rangle = 2\delta_{i,j}\delta(t-t')$ for $i, j = x, y, z$. $\langle\cdots\rangle$ denotes an ensemble average over the distribution of noise. $\delta(t)$ is the Dirac delta function. Imposing reflecting boundary conditions in the transverse direction ensures the confinement of the dynamics within the tube, while the periodic boundary conditions are enforced along the longitudinal direction. The tube shape is described by its radius

$$\omega(x) = a(1 - \cos x) + b, \tag{4}$$

where a is the parameter that controls the height of the tube and b is the radius at the bottleneck. $g(x)$ is the non-negative noise modulation with the same period as $\omega(x)$,

$$g(x) = \frac{1}{\sqrt{1 - \alpha\cos(x - \phi)}}, \tag{5}$$

where ϕ is phase shift between the noise modulation and the tube shape and α is the noise modulation amplitude. Since $g(x) > 0$ the amplitude α is restricted to the range $0 \leqslant \alpha < 1$.

The movement equation of Brownian particles moving along the axis of the 3D tube can be described by the Fick–Jacobs equation [15–17] which is derived from the 3D Smoluchowski equation after elimination of y and z coordinates by assuming equilibrium in the orthogonal directions. The reduction of the coordinates may involve not only the appearance of an entropic barrier but also the effective diffusion coefficient. When $|\omega'(x)| \ll 1$, the effective diffusion coefficient reads [15]

$$D(x) = \frac{D_0}{[1 + \omega'(x)^2]^{\gamma}}, \tag{6}$$

where $D_0 = k_B T/\eta$ and $\gamma = 1/2$ for 3D. The prime stands for the derivative with respect to the space variable x.

Consider the effective diffusion coefficient, the entropic barrier and a spatially modulated Gaussian white noise: the effective one-dimensional Langevin equation with the friction coefficient $\eta(x) = \frac{k_B T}{D(x)}$ reads in the Stratonovich prescription,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{A'(x)}{\eta(x)} - \frac{1}{2}k_B T g^2(x)\frac{\eta'(x)}{\eta^2(x)} + g(x)\sqrt{\frac{k_B T}{\eta(x)}}\xi_x(t), \tag{7}$$

where we define a free energy $A(x) := -TS = -Tk_B \ln h(x)$, $S = k_B \ln h(x)$ the entropy, $h(x)$ the dimensionless transverse cross section $\pi[\omega(x)/2\pi]^2$ of the tube in 3D. It should be noted that the above equation involves a multiplicative noise with an additional temperature-dependent drift term $(-\frac{1}{2}k_B T g^2(x)\frac{\eta'(x)}{\eta^2(x)})$ [18]. The additional term turns out to be essential in order for the system to approach the correct thermal equilibrium state. The motion is equivalently described by the Fokker–Planck equation [11–16],

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D(x)g^2(x)\frac{\partial P(x,t)}{\partial x} + \frac{D(x)}{k_B T}\frac{\partial A(x)}{\partial x}P(x,t) + D(x)g'(x)g(x)P(x,t)\right], \tag{8}$$

where $P(x,t)$ is the probability density for the particle at position x and at time t.

The stationary current J is determined by

$$J = -D(x)g^2(x)\frac{\partial P_{\mathrm{st}}(x)}{\partial x} - \frac{D(x)}{k_B T}\frac{\partial A(x)}{\partial x}P_{\mathrm{st}}(x) - D(x)g'(x)g(x)P_{\mathrm{st}}(x), \tag{9}$$

where $P_{\mathrm{st}}(x)$ is the steady probability density and satisfies the normalization condition $\int_0^{2\pi} P_{\mathrm{st}}(x)\,\mathrm{d}x = 1$ and the periodicity condition $P_{\mathrm{st}}(x) = P_{\mathrm{st}}(x + 2\pi)$. These lead to the general current formula [11–16],

$$J = \frac{1 - \exp(\frac{\Delta}{k_B T})}{\int_0^{2\pi} g^{-1}(x)\exp[-\frac{\Psi(x)}{k_B T}]\,\mathrm{d}x \int_x^{x+2\pi} D^{-1}(y)g^{-1}(y)\exp[\frac{\Psi(y)}{k_B T}]\,\mathrm{d}x}, \tag{10}$$

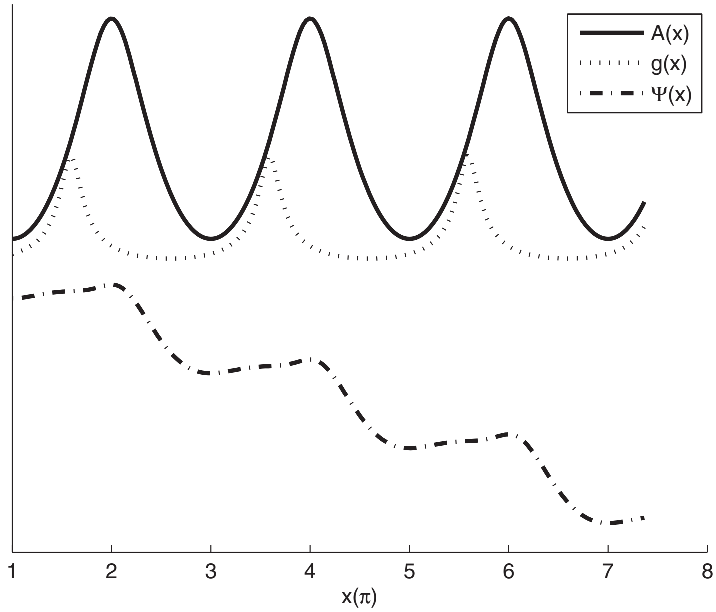

Figure 2. Free energy $A(x)$, noise modulation $g(x)$ and generalized potential $\Psi(x)$ as a function of x at $a = 1/2\pi$, $b = 0.5/2\pi$, $k_B T = 0.5$, $\alpha = 0.95$ and $\phi = 5$.

where $\Psi(x)$ and Δ are generalized potential and its slope from one periodicity, respectively:

$$\Psi(x) = \int_0^x \frac{A'(x)}{g^2(x)}\,\mathrm{d}x, \tag{11}$$

$$\Delta = \Psi(2\pi) = \frac{4\pi k_B T \alpha \sin\phi}{a}(a + b - \sqrt{2ab + b^2}). \tag{12}$$

The denominator of equation (9) is always positive, so the sign of J is determined by Δ. From equation (11), we can know that the sign of J is changed as periodic function of ϕ. It should be noted that the current will vanish when the amplitude of the spatial modulation α tends to zero. In this case, the multiplicative noise in the tube reduces to an additive noise and no current occurs.

3. Results and discussions

Figure 2 shows free energy $A(x)$, noise modulation $g(x)$ and generalized potential $\Psi(x)$ as a function of x at $\alpha = 0.95$ and $\phi = 5.0$. A phase shift between an entirely symmetric tube and noise modulation leads to a finite bias in the generalized potential equation (10). The physical origin of this bias is that a high noise intensity at one slope of free energy causes a higher escape probability compared to that at opposite slope. In figure 2, we use $\phi = 5.0$ and there is a bias in the generalized potential. Brownian particles in this case will go to the right on average.

Figure 3 shows the noise-induced current J as a function of α for $\phi = 5.0$. Figure 3 has been obtained from equation (9). When $\alpha = 0$, the noise modulation is a constant and the generalized potential is a periodic function, so the current goes to zero. The current increases with the noise modulation amplitude α.

Figure 4 shows the current J versus the bottleneck radius b for $\phi = 5$ and $\alpha = 0.95$. If the bottleneck is zero, the particle cannot pass through it, so the current tends to zero. When

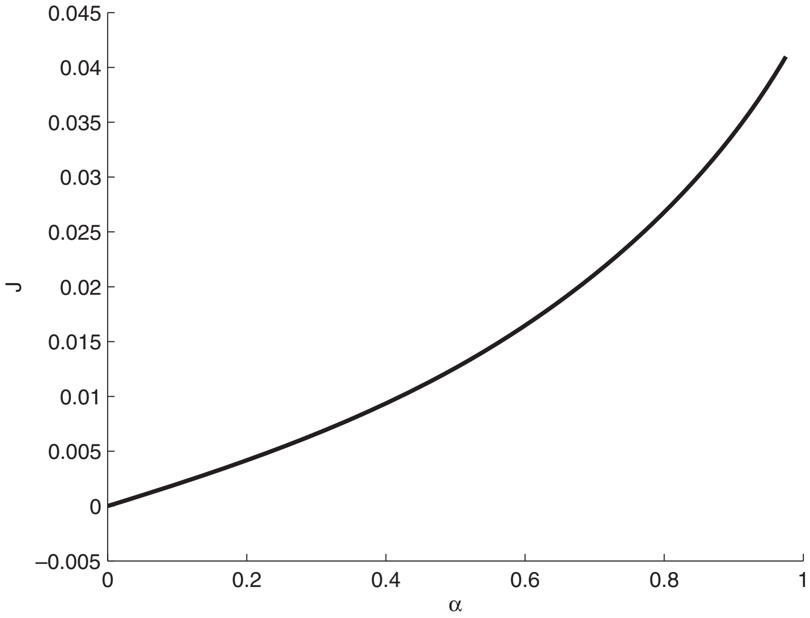

Figure 3. Current J versus noise modulation amplitude α at $a = 1/2\pi$, $b = 0.5/2\pi$, $k_B T = 0.5$ and $\phi = 5$.

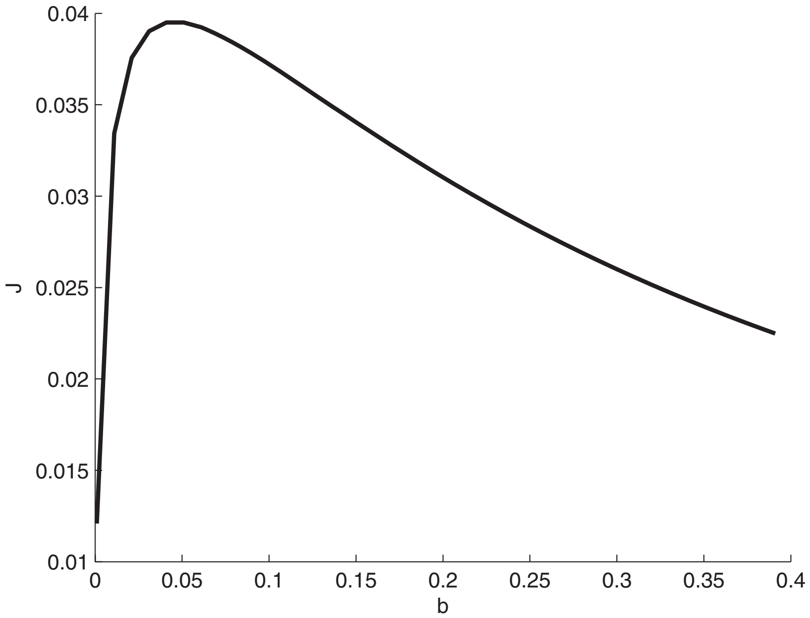

Figure 4. Current J as a function of the bottleneck radius b at $a = 1/2\pi$, $k_B T = 0.5$, $\alpha = 0.95$ and $\phi = 5$.

the radius at the bottleneck is infinite, the tube effect disappears and the current tends to zero, also. Therefore, there exists an optimized radius at which the current takes its maximum value.

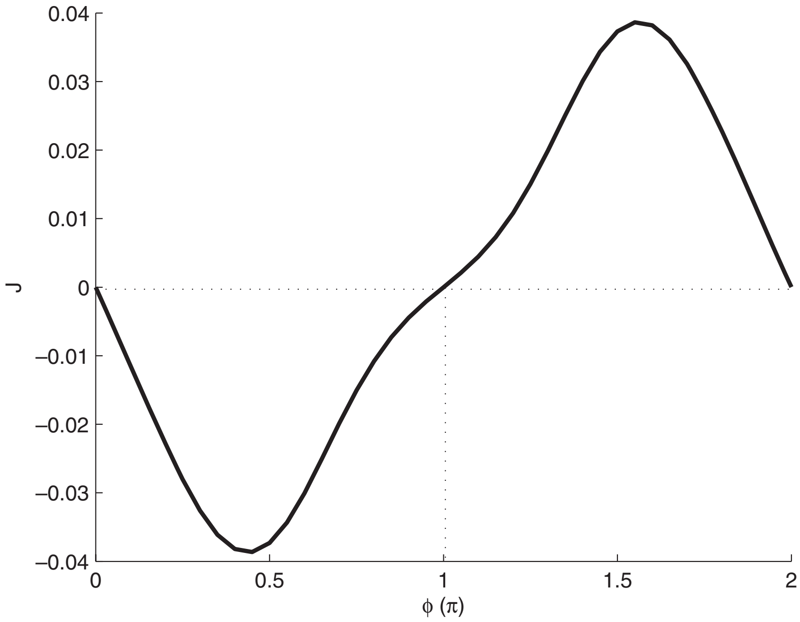

Figure 5. Current J as a function of the phase shift ϕ between the entirely symmetric tube shape and noise modulation at $a = 1/2\pi$, $b = 0.5/2\pi$, $k_B T = 0.5$, and $\alpha = 0.95$.

Figure 5 shows the current J as a function of phase ϕ. The figure is obtained by using equations (9)–(11). It is easy to obtain that the sign of the current is determined by ϕ. The phase ϕ plays an important role in obtaining a net current. If ϕ is a multiple of π the state-dependent noise only causes a redistribution of particles which is periodic with period 2π. If the phase ϕ is not a multiple of π then the noise intensity is asymmetric with regard to the local maximum of free energy. From equations (9)–(11), we can know that the current is a periodic function of ϕ with the period 2π. The current goes to zero at $\phi = 0$, π and 2π. The current is negative for $0 < \phi < \pi$ and positive for $\pi < \phi < 2\phi$.

Noise-induce transport occurs if and only if the generalized potential $\Psi(x)$ is not a periodic function [12], i.e. $\Psi(2\pi) \neq \Psi(0)$. A phase shift between the entirely symmetric tube and noise modulation can break the symmetry of the generalized potential. Similarly, the asymmetry of the tube and noise modulation can also induce a finite bias in the generalized potential. For example, we can consider the tube shape as asymmetric:

$$\omega(x) = a\left[\sin(x) + \frac{\Delta}{4}\sin(2x)\right] + b, \tag{13}$$

where Δ is the asymmetric parameter of the tube shape. The non-negative noise modulation is $g(x)$,

$$g(x) = \frac{1}{\sqrt{1 - \alpha \sin x}}, \tag{14}$$

where α is the noise modulation amplitude. The generalized potential $\Psi(x)$ is shown in figure 6. It is found that the asymmetric tube ($\Delta = -2$ and 2) and noise modulation (equations (13) and (14)) can break the symmetry of the generalized potential. However, the generalized potential is still symmetric for a symmetric tube ($\Delta = 0$) and noise modulation. Thus, the interplay between an asymmetric tube and noise modulation can also induce a net current.

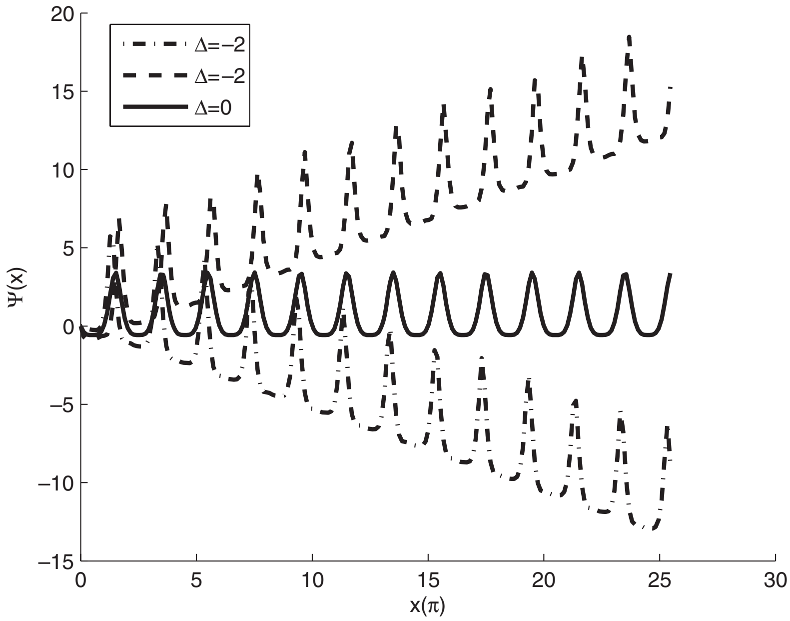

Figure 6. Generalized potential $\Psi(x)$ as a function of x at $a = 1/2\pi$, $b = 1.5/2\pi$, $k_BT = 1$, $\alpha = 0.95$ for different values of the asymmetric parameter $\Delta = -2, 0, 2$, respectively.

4. Concluding remarks

In this paper we consider Brownian particles moving in a 3D periodic tube subject to a spatially modulated Gaussian white noise. The reduction of the coordinates may involve not only the appearance of entropic barriers but also the efficient diffusion coefficient. We derive an analytical expression for noise-induced current by introducing entropic barriers. The phase shift between an entirely symmetric tube and noise modulation can induce a finite bias in the generalized potential. The physical origin of the net current is that the noise intensity is asymmetric with regard to the local maximum of free energy. Each local free energy hill has a slope with high-intensity noise (hot slope) and a slope with low-intensity noise (cold slope). Net current occurs because Brownian particles starting from the valley can climb the hot slope more easily than they can climb the cold slope. There exists an optimized radius at which the current takes its maximum value which is similar to that of the unbiased forced-driving periodic tube [16]. The phase ϕ plays a very important role in obtaining a net current. The sign of the current is determined by the phase ϕ. The current is a periodic function of ϕ with the period 2π. In a period, the current is negative for $0 < \phi < \pi$ and positive for $\pi < \phi < 2\pi$. It is also found that the asymmetric tube and noise modulation can break the symmetry of the generalized potential and induce directed transport.

Acknowledgments

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